Data Locality in Parallel Rendering

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Abstract. One of the main challenges in global illumination is rendering scenes with millions of polygons and megabytes of textures. Combining the processing power and the memory of multiple processors or workstations to render these complex scenes is an attractive proposition but the complex interactions between data and processing introduces a significant amount of data communication. Data locality methods may improve cache coherence and cache access coherence by finding an optimal data partitioning, by re-ordering computations, and by replacing complex geometry with simplified image-based representations. We review the different data locality methods and focus on local caching of global radiance values. We present the results of an implementation in the ray tracing program Radiance.

1 Introduction

Parallel rendering has long been considered an answer to the high computational demands of global illumination algorithms such as ray tracing and radiosity, and the ever increasing size of models has recently spurred a renewed interest in this subject. The complexity of computer models has increased tremendously over the years due to improved data sampling and more powerful modelling techniques. Scenes with several millions of polygons and many megabytes of texture images are no exception anymore. Besides model data, most algorithms also employ additional data structures such as radiosity meshes, spatial subdivision structures, mipmaps, and other methods to increase the efficiency and quality of the sampling. These temporary data structures may also take many megabytes. Given the total memory requirements of several gigabytes, distributing data in addition to distributing processing, may well prove to be a necessity in such demanding applications.

Distributing the processing and the data over multiple processors, however, creates an overhead in the form of communication needed to match processing and data. Either tasks are migrated to processors that store the necessary data (data parallel approach) or tasks are distributed and data is communicated on request (demand-driven approach). The data parallel approach is scalable with respect to problem size and can render arbitrarily large models, but it is difficult to spread the processing evenly over the processors, due to 'hot spots' in the scene, such as light sources, and due to the uneven distribution of processing over time. Dynamic adaption of the data distribution over the processors is complex and introduces extra overhead.

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The demand-driven approach achieves a better load balance but the efficiency of this approach is very much dependent on the amount of coherence that can be found in subsequent data requests. Assuming that data retrieved from other processors is cached locally, in highly complex scenes coherence between subsequent data request will be low and therefore caching is of less use. Hybrid approaches try to combine data-parallel and demand-driven scheduling in such a way that non-coherent data tasks are scheduled data parallel and data coherent tasks are scheduled demand-driven [32, 14, 25]. The demand-driven tasks then compensate for workload variations in the data parallel task distribution. However, when the scene is complex and the amount of available data coherence is low, the hybrid scheduling suffers a loss of suitable demand-driven tasks and falls back to the (in-)efficiency of the data-parallel approach.

It turns out that for both the demand-driven approach and the data parallel approach, the efficiency is very much dependent on the amount of data locality that can be achieved. When it is not possible to partition the data and tasks such that requests for remote data are minimal, then additional methods are needed to reduce data or task communication, for instance by locally representing remote data in a compact format or by caching intermediate results. In this paper we will review some of these methods and discuss their application to parallel rendering. We will do this in the context of a ray tracing algorithm using diffuse sampling, such as described in [39], but the methods are applicable to other ray tracing and radiosity algorithms as well. Data locality techniques are also useful in single processor configurations to reduce the amount of data transfers between the different levels of caches, memories and discs [23].

The paper is structured as follows. We first discuss several methods to improve data locality such as data partitioning, access coherence, impostor techniques and (ir-)radiance caching (section 2). Then we present some experiments with incoming radiance caching (section 3) and draw some conclusions (section 4).

2 Data Locality

Data locality is a measure of how well data can be selected, retrieved, compactly stored, and re-used for subsequent processing tasks. Finding a good partitioning to distribute data is a first step to successfully associate data with specific tasks. A next step is to optimise access coherence, i.e. to find an optimal ordering of the computations to minimise the number of times data items will have to be re-communicated or re-read from disc. Alternatively, data can be compressed or replaced by a more compact representation, in particular for remote objects and environments, when an exact representation is not required.

2.1 Data Partitioning

When data has to be distributed, a first issue is how scene data should be divided over the processors. Initial data distributions can have a strong impact on how well the system performs. The more evenly the workload associated with the data is distributed, the less idle time is to be expected. In order to be able to distribute a part of the scene, for instance a cell of an octree spatial subdivision, first the expected cost per voxel should be computed. In a second step, the voxels can be distributed across the processors, preserving data locality as much as possible, while at the same time attempting to equalise the cost per processor.

Some initial research has proved that it is possible to estimate the cost of a single ray traversing an octree structure [17, 36, 27]. The number of intersection tests for each node in the octree can be estimated by averaging the depth of the leaf cells of the octree, weighting these depths by the surface area of the cells, and computing the probability that a ray traversing that voxel intersects one of the objects contained within the voxel. Alternatively, the cost per voxel could be predicted by estimating the number of rays that would traverse each voxel during rendering [26]. Such an algorithm would take into account the distribution of objects over the scene, as well as the view point and the position of light sources. The data distribution would therefore be tailored to the particular view point chosen.

Given an initial load estimation, a data distribution may be devised such that as much object coherence as possible is retained. A possible algorithm to distribute octree branches over processors may use the cost per voxel information to split off separate branches. The nearer to the root of the tree this split occurs, the more coherence between sub-parts of the scene is preserved. However, it should be noted that preserving coherence and equalising the estimated workload per processor are conflicting goals.

A different and slightly more complicated algorithm is called region growing [38], where each processor receives a seed voxel. At each subsequent step, a voxel adjacent to one of these seed voxels is assigned to the same processor. The estimated workload is distributed evenly by adding the adjacent voxel with the highest cost function to the processor with the lowest accumulated cost function. This process is continued until all voxels are assigned to a processor.

Octrees and grids are axis-aligned subdivisions that do not always segment a model in an optimal way. More data coherence may be obtained with a space partitioning that is aligned with the internal structure of the model. The BSP-tree method uses planar surfaces of the model to derive a binary space partition. Several strategies have been developed to choose the most suitable faces as primary dividing planes and to optimise the data coherence within the resulting space partitions (preferably cubic and not long and thin) while keeping the number of split surfaces as low as possible [5, 21, 37]. In [20] a clustering algorithm is applied to the surfaces in a building environment to align the partitioning with the structure of the floor plan.

A very fruitful approach is to partition the scene on the basis of inter-visibility [37]. Energy transfer can only take place between surfaces that are mutual visible. A visibility preprocessing can be used to create local clusters and to calculate visibility between these clusters. This information can be stored in a visibility graph with the clusters as nodes. The importance of energy transfer between surfaces can be taken into account such that two surfaces with a high 'form-factor' interaction are likely to end up in the same cluster [6].

2.2 Access Coherence

Cache coherence may be improved by re-ordering computations, e.g. by temporarily suppressing computations that need data which is locally not available, or by grouping tasks that share similar data needs. Deviating from the sequential order will always introduce some extra storage of state information. This is not uncommon in parallel implementations where tasks are migrated or tasks are temporarily suspended waiting for data to arrive. Re-ordering may be accomplished for instance by grouping shadow rays or by performing ray tracing in breadth-first order [12]. Grouping primary rays together allows an efficient selection of data and guarantees a high cache access coherence [24, 33]. In fact, any multi-pass or pre-processing may be viewed as a re-ordering of computation (compared to a full MC ray tracing algorithm). A separate light pass may significantly increase ray coherence at the cost of the storage of a photon map or radiance cache.

Finding an optimal traversal in the visibility graph to minimise the number of cluster swappings during one interaction of a progressive radiosity pass can be approached as a travelling salesman problem [19]. An other scheduling strategy is to use the voxels of a spatial subdivision as a 'scheduling grid' [23, 22]. In this approach, rays are processed in voxel order. A voxel with object data is only loaded (or created) when enough work is available or when its contribution to the final rendering wins over processing other data voxels. The advantage is that rays that pass through the same voxel are processed together, even if their origin or descend is different.

Beside the cost for storing extra state information, there may be a penalty involved for not taking the optimal order of computations. For instance, deferring some shooting tasks in progressive radiosity may lead to a slower convergence [6].

2.3 Impostor Techniques

Even with a suitable data partitioning and coherent cache access, interaction between data partitions may not be completely avoided, leading to either data or task communication. The amount of data communication can be reduced by replacing the remote data with some local 'placeholder'. This may be a simplified representation or an image 'impostor' of the real data. One of the first applications of this idea was the introduction of so-called 'virtual walls', separating planes between partitions placed at strategic places (e.g. portals) to serve as interface between two partitions [41, 2]. A drawback is that when only radiosity is transferred through the virtual walls, directional information will be lost.

Virtual interfaces [1, 29] borrow the data partitioning from the virtual walls technique but are an improvement in that directional information is maintained. Each source patch first shoots its energy into its own environment. Then it is passed to the neighbouring partitions together with a visibility (or shadow) mask. This mask is a discretisation of the hemisphere around the source patch. The directions that are blocked by objects in the processed environments are disabled. The data transferred to the neighbouring partitions is thus exact in position and size but is approximate in the discretisation of the directions left open after blocking of the light by intermediate geometry. A similar technique using the convolution of a blocking mask and an incoming light source to project shadows onto surfaces is proposed in [35].

Environment mapping [3, 10] is an other variant on the virtual walls technique. Each partition or object cluster is surrounded by an environment map to represent the incoming intensity from the rest of the scene. Instead of fetching data from remote processors, or migrating tasks to other processors, a simple table lookup is enough to find

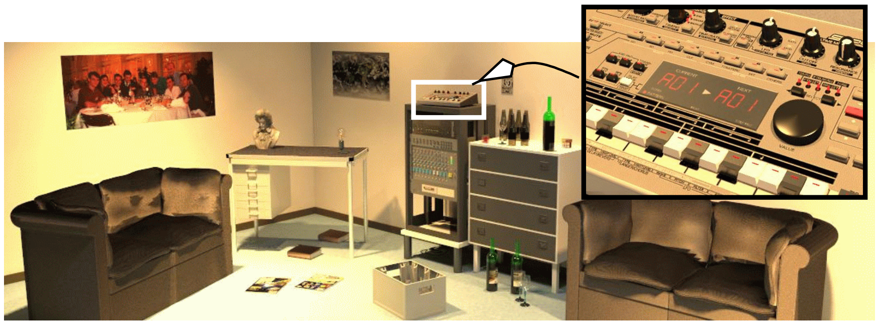


Fig. 1 Clusters of small objects, such as the synthesiser enlarged in the inset, may be fetched at a lower resolution for the chosen viewpoint.

an approximate shading value which would otherwise be very costly to compute [28]. As the environment map is created from only one arbitrary view point within the partition and used for any ray origin in this partition, it will introduce errors which show up in the form of displaced reflections. When used solely for diffuse interreflection, this may however not prove to be much of a problem. The accuracy can be controlled by increasing the radius of the environment map with respect to the size of the local partition.

Geometric simplification is another successful strategy. The basic idea is that if a complex object or a densely populated part of the scene is at some distance, then without sacrificing accuracy, the complex geometry can be replaced with a simplified geometry carrying the same energy and having similar reflection and emission properties.

An example where the full resolution of geometry data is not always required is given in figure 1. Assuming that the objects are distributed across a number of processors, the processors responsible for the area where the viewpoint is, may occasionally need to access data that is far away. The knobs on the equipment at the back of the room could be fetched at a far lower resolution without affecting the quality of the sampling.

There are many different ways in which geometry can be simplified or stored at different resolutions. The methods that should be considered are ones where geometry is replaced with simpler geometry and techniques where geometry is replaced with shading information (impostors, grouping of patches).

Geometric simplification algorithms attempt to reduce the polygon count of objects by replacing large groups of small surfaces by a small group of larger surfaces [13, 8]. Such algorithms usually do not replace different surface reflectance properties by an average BRDF. However, preserving average material properties is important for geometric simplification algorithms to be useful in realistic rendering algorithms.

This issue was addressed by some of the grouping and clustering techniques developed for hierarchical rendering. A locally dense occupation of small polygons, e.g. a plant, can be replaced by an enclosing volume that inherits the same global shape, reflection, emission, absorption or transparency properties as the original. Several methods have been developed differing in the way they represent these properties, either as shading transfer [15], transmission transfer [34] or reflection distribution [31]. Most of these techniques where originally developed to improve the sampling quality, as the intersection of a complex object is not only expensive but also prone to aliasing.

Textures can be used to replace detail geometry, e.g. to represent ornamental decorations on a façade. If the geometry is simplified to a single polygon or box and the texture represents the complete geometry then we talk about image impostors [7, 18]. Image impostors work in much the same way as environment maps, the difference being that rather than replacing the surroundings of an object with a box plus texture map, the object itself is replaced with a simple cube plus texture map. Therefore, impostors borrow from environment mapping in construction and from geometric simplification in usage.

Image-based rendering techniques may also be viewed as impostor techniques, as they can represent the incoming radiance from a scene for a certain region of view points and directions. However, the storage costs of these techniques are still prohibitive.

2.4 Irradiance Caching

The disadvantage with impostor techniques is that they all require a pre-processing stage, and sometimes even require user intervention, which makes them slightly awkward to use. A possible method to overcome this, is to build a data structure on the fly. For instance, when a processor holds part of the scene and for reflection calculations has to sample its environment, then either ray tasks are submitted to other processors or remote data is requested, but in both cases the query is expensive. Storing the results of these queries allows us to re-use this information for rays in similar directions. Thus for subsequent rays the data structure is queried. If there is sufficient information present in the data structure, this is used (as in environment mapping). If not, the ray task is executed as before and when a shading result is returned, it is used for both shading the ray and updating the data structure. The further the computation proceeds, the less communication between processors will be necessary, as processors will build up an image of what surrounds them. We term such methods "directional irradiance caching" [9]. The following section describes an implementation of this technique for the Radiance program.

A suitable data structure to store such incoming radiance values is a 5D tree, adaptively subdivided both on position and direction. Each node is subdivided when the number of averaged radiance values exceeds a certain threshold. Such a tree structure has been used to predict important sampling directions and therefore minimises the amount of time spent on directions that do not substantially contribute to the final shading [16].

Instead of storing the incoming radiance (or field radiance function), also the resulting irradiance values for all directions could be stored. Irradiance at a point on a surface is the cosine-weighted integration of the incoming radiance over all directions of the hemisphere. In case of diffuse reflection the radiance at that point is then easily computed by multiplying the irradiance with ρ/π. The irradiance distribution function is much smoother than the radiance distribution function and can be more compactly represented [11]. However, it requires a preprocessing as it is more complex to adaptively built this representation (unless all incoming radiance values are stored as well).

Other methods could be borrowed from image-based rendering techniques with re-

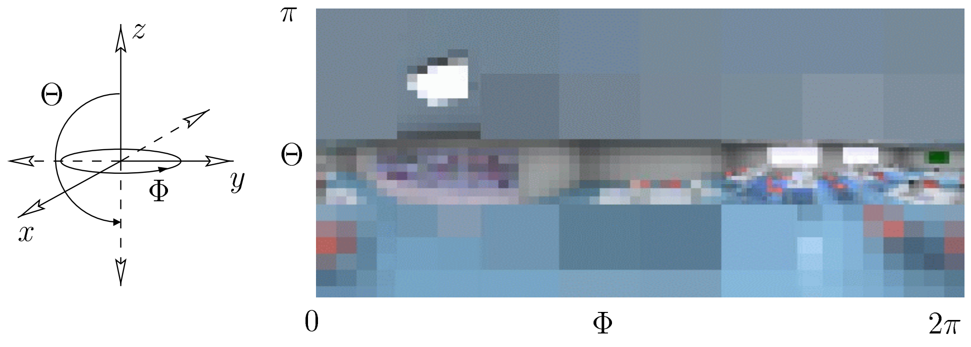


Fig. 2 The surroundings of a voxel are discretised into a full sphere using a quadtree. The right image shows what a fully developed cache may look like for the voxel shown in figure 3.

spect to compact representation of incoming radiance values from different directions and for a specified region of view points.

3 Directional Irradiance Caching

In this section we describe some experiments with an implementation of a directional irradiance cache within the Radiance lighting simulation package [39].

3.1 Radiance

The ray tracing method employed in the Radiance package is an extended form of ray tracing which also samples the diffuse interreflection in a scene. This diffuse reflection at a point is calculated by constructing a hemisphere over the point and sampling a large number of directions over this hemisphere. The number of rays to accurately sample the diffuse interreflection can be very high. To reduce the number of rays Radiance uses two methods, an adaptive sampling method to concentrate the sampling effort in directions with a high variance, and a radiance cache to store irradiance values computed earlier [40]. When a new intersection is found and if sufficient irradiance values are stored in its vicinity, instead of sampling the diffuse reflection, a new irradiance estimate is obtained by interpolating some of the cached irradiance values. The validity of this principle is based on the fact that diffuse reflection tends to vary slowly over a surface. Moreover, the error introduced by the interpolation can be kept within bounds by specifying a maximum radius in which the cached value is valid.

The directional (incoming) radiance cache method that we propose can be used in conjunction with the above interpolation method. Instead of storing the end result of the hemisphere sampling, it stores the incoming radiance values as sampled by the diffuse rays.

3.2 Directional Cache

In our implementation we associate one cache with each leaf node of an octree spatial subdivision. The directional cache is created on the fly and is adaptively subdivided in latitudinal and longitudinal directions, resulting in a quadtree tree structure (figure 2). Each node of the tree represents a solid angle and stores an average intensity, the number of rays (samples), and a variance value. Each leaf node is subdivided whenever its 'samples' value equals a specified minimal number. This way, we ensure that the average intensity stored in a leaf node is mainly composed of intensity values obtained by rays traced within the leaf node's own solid angle [1]. When the solid angle covered by a node is small enough, the node steps into a second stage. In this stage leaf nodes in the tree are subdivided using variance as a criterion. The reliability of the stored variance depends on the number of samples from which it was calculated. For this reason, a leaf node is subdivided only when it contains enough samples and the variance between these samples is considered high. Node subdivision is constrained by a maximum depth. When a given cache partition is sufficiently sampled, no further sampling in that direction will be done, and the cache values can be re-used for subsequent rays that are sampled in that direction.

However, just like the environment map, we do not use the cache for rays that find an intersection within a specified radius from the centre of the node. As the rays' origins may be distributed over the cell, the cache will not give an accurate prediction for rays that find an intersection nearby. The cache therefore is only used to represent the environment at a certain distance of the node. Local geometry - which in general will have a significant impact - is always sampled accurately.

Tracing diffuse rays is thus performed in stages. Each ray is traced locally, until either an intersection is found, or the ray leaves the local neighbourhood (see figure 3). If the ray is not considered to be local anymore, the directional cache is read to determine whether the information in the cache is accurate enough for the ray's direction. If this is the case, the directional cache provides an estimate for the shading result of the ray. Otherwise, the ray needs to be traced.

Controlling the size of the radius with respect to the size of the voxel provides a convenient way of trading accuracy for speed.

3.3 Implementation and Results

In order to measure the effectiveness of the directional cache, the cache was implemented in the Radiance program and used in combination with Radiance's own radiance cache. Several tests were performed using different quality settings, in line with values recommended for Radiance. A high quality setting was chosen to serve as a reference. The scene used to evaluate the directional cache with these settings, is the well-known conference room, as depicted in figure 4. All images were rendered at a resolution of 512 x 512, and filtered to a resolution of 256 x 256.

Table 1 shows the reduction in number of intersection tests (performance) and rendering time, obtained with the directional cache. A mean squared error (MSE) was

[1]Upon subdivision, only the average intensity of the parent node is copied, the 'samples' value is set to one.

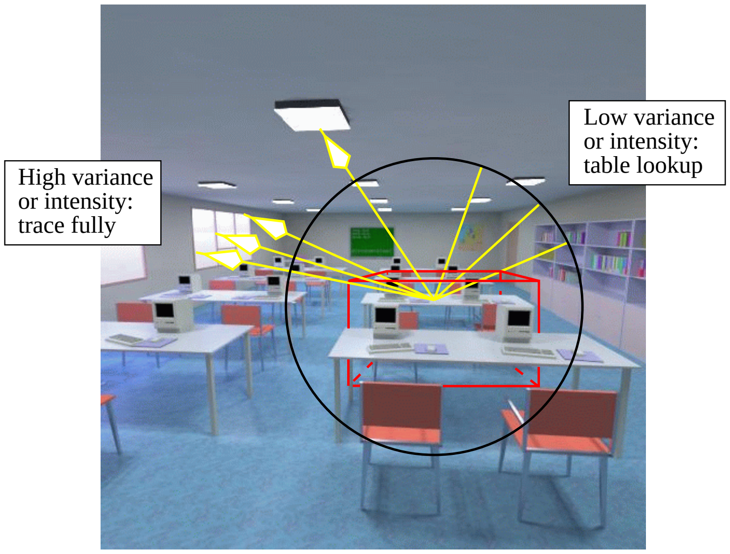


Fig. 3 When reaching the perimeter of locality without intersecting an object, only in important directions the sampling is continued. The remainder of the rays are estimated by reading the cache.

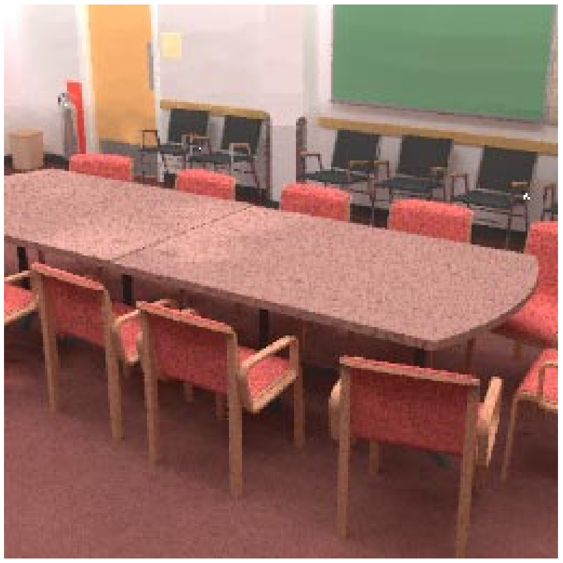


Fig. 4 The Conference Room.

Quality	Radiance		Directional Cache			
	Intersection Tests	Time	Intersection Tests	Time	Gain	
Accurate	257,103,496	16.6	222,453,281	14.2	13.5%	14.8%
Fast	119,065,157	5.4	115,040,115	5.1	3.4%	5.8%
Low	106,789,984	4.4	104,533,101	4.3	2.1%	3.8%

Table 1 Performance, counted both in total number of intersection tests and hours of calculation time needed. The percentages in the last column represent the performance gain in number of intersection tests and time respectively. The reference image took 714,815,976 intersection tests to render.

Quality	Radiance	Directional Cache
Accurate	3.37	4.78
Fast	7.10	8.45
Low	9.59	12.36

Table 2 Quality comparison for each of the quality settings using the Daly method. The numbers give the mean squared error (MSE) for each of the rendered images, compared with the reference image.

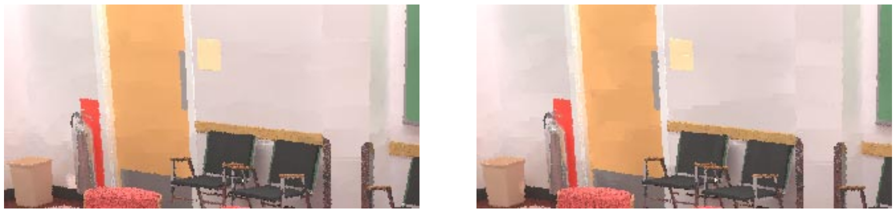
Quality	Directional Cache				Cache hit
	Diffuse rays	Local	Global	Lookup	
Accurate	4,549,780	2,044,615	982,182	1,522,983	60.8%
Fast	573,027	214,636	191,406	166,985	46.6%
Low	223,889	63,849	105,828	54,212	33.9%

Table 3 Number of rays shot for each of the three quality settings.

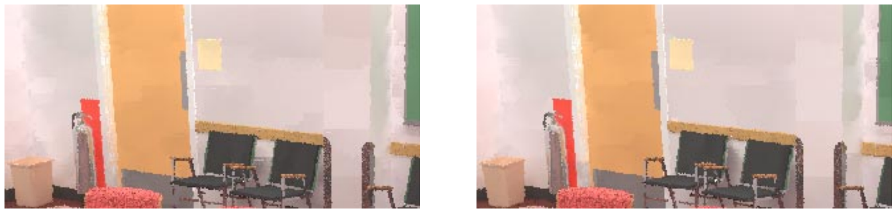
computed based on the perceptional Daly model [4, 30], to give an indication of how the images perceptually differ from the reference image. Table 2 shows the results of this quality comparison, and figure 5 shows the actual Radiance and Directional Cache images. Table 3 gives the total number of diffuse rays traced for the conference room model. This number is subsequently split into a component which counts the number of rays that intersect with a nearby object, the number of rays that are traced outside the local perimeter (because the cache was not sufficiently filled for that direction) and the number of rays that could be approximated by a cache lookup.

The cache hit ratio is defined as the number of cache lookups over the number of rays that reached the perimeter (global plus lookup). As can be seen, the cache gives a reduction of up to 60% in the number of global rays, which for a parallel implementation would greatly reduce the communication costs. The MSE figures show that the directional cache looses some of the accuracy obtained with standard Radiance. Visual artifacts, however, are hardly noticeable, which may be partly due to the linear interpolation of Radiance's own cache.

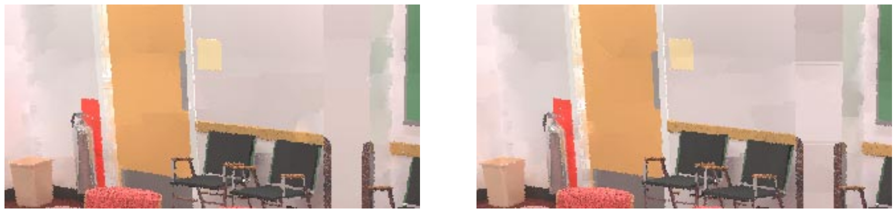
Beside the reduction in global rays, the method has several other advantages. Like the environment mapping algorithm, the cache builds a 'filtered' representation of the environment and reduces the over-sampling of far-away areas that do not contribute that much. Moreover, the reduction in global rays is for the bigger part realised towards the end of the rendering process, which is advantageous in parallel rendering, because non-local rays cause long termination delays [25].



‘Accurate’ quality settings.



‘Fast’ quality settings.



‘Low’ quality settings.

Fig. 5 Detail of images rendered using standard Radiance (left) and the Directional Cache (right).

4 Conclusion

Model complexity has become a primary motivation to use distributed processing. But to achieve some efficiency and scalability we need data locality methods that optimally partition the data and that compactly represent remote data. In the context of rendering, this often means conversion of geometry and lighting into radiance information. In this paper we have given an overview of data locality methods for parallel rendering and we have focused on a directional cache to store and re-use incoming radiance values. We have presented the results of an implementation in the program Radiance. Although the number of non-local rays is drastically reduced, the overall effect on the cache efficiency and the data communication in a parallel implementation still has to be assessed.

Acknowledgements

All images were rendered using the Radiance lighting simulation package [39]. The authors would like to thank Greg Ward Larson for putting his programme and the conference room model in the public domain. Thanks to Arjan Kok for allowing us to use the results of his modelling efforts. Rick Germs did the implementation and the testing of the irradiance cache. We also thank Alan Chalmers for providing useful comments. This work was funded in part by the EC under TMR grant number ERBFM-BICT960655.

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